

$$\text{Volume of the water displaced} = \frac{0.02}{1000} \\ = 2 \times 10^{-5} \text{ m}^3$$

$$\text{Volume of stone} = 2 \times 10^{-5} \text{ m}^3$$

(b) ~~The reading on the spring balance~~ (Take $g = 10 \text{ m/s}^2$ and density of water 1000 kg/m^3) ~~The density of the stone~~

Solution

$$\text{Mass of stone} = \frac{2.0}{10}$$

$$= 0.2 \text{ kg}$$

$$\text{Density of the stone} = \frac{\text{mass of stone}}{\text{Volume of stone}}$$

$$\frac{0.2 \text{ kg}}{2 \times 10^{-5} \text{ m}^3} \\ = 2500 \text{ kg/m}^3$$

4 A meteorological balloon has a volume of 36 m^3 and is filled with helium of density 0.18 kg/m^3 . If the weight of its fabric is 120 N , calculate the maximum load which the balloon can lift. (Take density of air as 1.3 kg/m^3)

Solution.

$$\text{Volume of air displaced by the balloon} = 36 \text{ m}^3$$

$$\text{Mass of air displaced by the balloon} = (36 \times 1.3) \text{ kg}$$

$$\text{Weight of air displaced} = 36 \times 1.3 \times 10 \\ = 468 \text{ N}$$

$$\text{Weight of air displaced} = \text{Upthrust} \\ = 468 \text{ N}$$

$$\text{Mass of helium in the balloon} = (36 \times 0.18 \times 10) \text{ kg} \\ = 64.8 \text{ kg}$$

$$\text{Weight of helium in the balloon} = (36 \times 0.18 \times 10 \times 10) \\ = 64.8 \text{ N}$$

$$\text{Weight inflated balloon} = (64.8 + 120) \text{ N} \\ = 184.8 \text{ N}$$

$$\text{Upthrust} = \text{Maximum Load} + \text{weight of inflated balloon}$$

$$\text{Maximum load} = \text{upthrust} - \text{weight of balloon} \\ = 468 - 184.8 \\ = 283.2 \text{ N}$$

Law of Floatation - It states that a floating object displaces its own weight of fluid in which it floats.

Experiment

To investigate the upthrust on a floating object

Apparatus

Measuring cylinder, water, test tube, sand, Weighing balance.



Procedure

- Half fill the measuring cylinder with water and record the level.
- Place a clean dry test tube into the cylinder and add some sand to it so that it floats upright.
- Record the new water level.
- Determine the volume of water displaced.
- Remove the test tube from the cylinder, dry it and determine its weight.
- Repeat the experiment four times, adding a little more sand each time.
- Record the results.

Observation

- The test tube sinks deeper with each addition of sand.
- The weight of the test tube and its contents is equal to the weight of water displaced.

Conclusion

A floating object displaces its own weight of the fluid in which it floats.
This is the law of floatation.

Examples

1. A boat of mass 1000 kg floats on fresh water. If the boat enters sea water, determine the load that must be added to it so that it displaces the same volume of water as before (take density of fresh water as 1000 kg/m^3 and density of sea water as 1030 kg/m^3).

Solution

$$\begin{aligned}\text{Weight of the boat} &= 1000 \times 10 \\ &= 10,000 \text{ N}\end{aligned}$$

By the law of floatation

$$\begin{aligned}\text{Weight of fresh water displaced} &= \text{Weight of the boat} \\ &= 10,000 \text{ N}\end{aligned}$$

$$\begin{aligned}\text{Mass of fresh water displaced} &= \frac{10,000}{10} \\ &= 1000 \text{ kg}\end{aligned}$$

$$\text{Volume of fresh water displaced} = \frac{1000}{1000} = 1 \text{ m}^3$$

$$\text{Volume of sea water displaced on addition of load} = 1 \text{ m}^3$$

$$\begin{aligned}\text{Mass of sea water displaced} &= 1 \times 1030 \\ &= 1030 \text{ kg}\end{aligned}$$

$$\begin{aligned}\text{Weight of sea water displaced} &= (1030 \times 10) \\ &= 10,300 \text{ N}\end{aligned}$$

$$\begin{aligned}\text{Extra load required} &= \text{Weight of sea water displaced} - \text{Weight of fresh water displaced} \\ &= 10,300 - 10,000 \\ &= 300 \text{ N}\end{aligned}$$

2. A balloon of volume 6.0 m^3 is filled with hydrogen of density 0.09 kg/m^3 and floats in air of density 1.3 kg/m^3 . Calculate the weight of the fabric of the balloon.

Solution

$$\begin{aligned}\text{Mass of hydrogen in the balloon} &= 6.0 \times 0.09 \\ &= 0.54 \text{ kg}\end{aligned}$$

$$\text{Weight of hydrogen} = 0.54 \times 10 \\ = 5.4 \text{ N.}$$

Let W be the weight of the fabric of the balloon.

$$\text{Total weight of the inflated balloon} = (5.4 + W) \text{ N.}$$

$$\text{Volume of air displaced by the balloon} = 6.0 \text{ m}^3$$

$$\text{Mass of air displaced} = 6.0 \times 1.3 \\ = 7.8 \text{ kg}$$

$$\text{Weight of the air displaced} = 7.8 \times 10 \\ = 78 \text{ N}$$

$$\text{By the Law of flotation, } 5.4 + W = 78$$

$$W = 78 - 5.4 \\ = 72.6 \text{ N.}$$

3 A cube of side 4 cm weighs 1.12 N in air. Calculate

a) its apparent weight when immersed in a liquid of density 0.79 g/cm^3

Solution

$$\text{Volume of the cube} = 4 \times 4 \times 4 \times 10^{-6} \\ = 6.4 \times 10^{-5} \text{ m}^3$$

$$\text{Volume of a liquid displaced by the cube} = 6.4 \times 10^{-5} \text{ m}^3$$

$$\text{Mass of liquid displaced} = (6.4 \times 10^{-5} \times 7.9 \times 10^2) \\ = 5.06 \times 10^{-2} \text{ kg}$$

$$\therefore \text{Weight of a liquid displaced} = 5.06 \times 10^{-2} \times 10 \\ = 5.06 \times 10^{-1} \text{ N}$$

$$\text{But weight of liquid displaced} = \text{upthrust on cube}$$

$$\therefore \text{Apparent weight} = 1.12 - 0.506 \\ = 0.614 \times 10^{-1} \text{ N.}$$

b) The density of the material of the cube

Solution

$$\text{Weight of the cube} = 1.12 \text{ N}$$

$$\therefore \text{Mass of the cube} = \frac{1.12}{10}$$

$$= 1.12 \times 10^{-1} \text{ kg}$$

$$\text{Volume of the cube} = 6.4 \times 10^{-5} \text{ m}^3$$

$$\text{Density of the cube} = \frac{1.12 \times 10^{-1}}{6.4 \times 10^{-5}}$$

(Sne)

$$= 1.75 \times 10^3 \text{ kg/m}^3$$

4 A model ferry boat 50 cm long and 20 cm wide floats in fresh water. If the ferry sinks 6 cm as a result of loading, calculate the load on the boat

Solution

$$\text{Volume of water displaced as a result of loading} = 6 \times 50 \times 20 \times 10^{-6} \\ = 6 \times 10^{-3} \text{ m}^3$$

$$\text{Mass of water displaced} = 6 \times 10^{-3} \times 1000 \\ = 6 \text{ kg}$$

$$\text{Weight of water displaced} = 6 \times 10 \\ = 60 \text{ N}$$

$$\text{Extra load on the model} = \text{Weight of extra water displaced} \\ = 60 \text{ N}$$

- 5 A balloon has a volume 100m^3 . If the weight of the balloon fabric is negligible, determine the maximum load it can lift when filled with hydrogen (Take density of air $= 1.2\text{kg/m}^3$, density of hydrogen $= 0.09\text{kg/m}^3$)

Solution

Volume of hydrogen in the balloon $= 100\text{m}^3$

$$\text{Mass of hydrogen} = 100 \times 0.09 \\ = 9.0\text{Kg}$$

$$\text{Weight of hydrogen} = (9.0 \times 10)\text{N}$$

Volume of air displaced by the balloon $= 100\text{m}^3$

$$\text{Mass of air displaced} = (100 \times 1.2)\text{Kg}$$

$$\text{Weight of air displaced} = 1.2 \times 10^3 \times 10 \\ = 1.2 \times 10^3\text{N}$$

But upthrust $=$ Weight of air displaced

$$\text{Upthrust on the balloon} = 1.2 \times 10^3\text{N}$$

Maximum load possible $=$ Upthrust $-$ Weight of hydrogen in the balloon

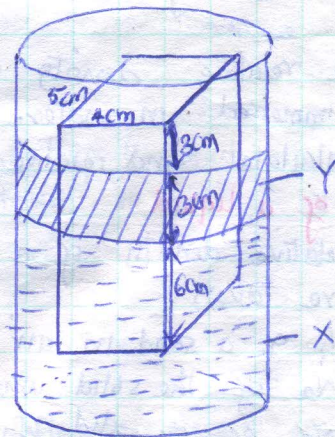
$$= 1.2 \times 10^3 - 9.0 \times 10$$

$$= 1.11 \times 10^3\text{N}$$

- 6 The wooden block floats in two liquids X and Y. Given the densities of X and Y are 1g/cm^3 and 0.8g/cm^3 respectively, determine:

(a) The mass of the block

Solution



Solution

$$\text{Volume } V \text{ of liquid Y displaced} = 5 \times 4 \times 3 \times 10^{-6}\text{m}^3 \\ = 6 \times 10^{-5}\text{m}^3$$

$$\text{Upthrust} = \rho V g$$

$$= 800 \times 6 \times 10^{-5} \times 10$$

$$= 0.48\text{N}$$

$$\text{Volume } V \text{ of liquid X displaced} = 5 \times 4 \times 6 \times 10^{-6}\text{m}^3 \\ = 1.2 \times 10^{-4}\text{m}^3$$

$$\text{Upthrust} = \rho V g$$

$$= 1000 \times 1.2 \times 10^{-4} \times 10$$

$$= 1.2\text{N}$$

$$\text{Total upthrust on block} = 0.48 + 1.2$$

$$= 1.68\text{N}$$

The block is floating, hence weight of block equals total upthrust

∴ Weight of block = 1.68 N

Mass of block = $\frac{1.68}{10}$

$$= 0.168 \text{ kg}$$

(b) The density of the block

Volume of block = $5 \times 4 \times 12 \times 10^{-6} \text{ m}^3$

$$= 2.4 \times 10^{-4} \text{ m}^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{0.168}{2.4 \times 10^{-4}}$$

$$= 700 \text{ kg/m}^3$$

Upthrust and Relative density

Relative density of a solid = $\frac{\text{Mass of the solid}}{\text{Mass of equal volume of water}}$

Since mass is directly proportional to weight, the relative density can also be expressed as

$$\begin{aligned} \text{Relative density of a solid} &= \frac{\text{Weight of solid}}{\text{Weight of equal volume of water}} \\ &= \frac{\text{Weight of solid}}{\text{Weight of water displaced by solid}} \end{aligned}$$

But weight of water displaced by solid = Upthrust

$$\therefore \text{Relative density of a solid} = \frac{\text{Weight of solid}}{\text{Upthrust in water}}$$

- To determine the relative density of a solid, the solid is weighed in air and then totally immersed in water.

The upthrust is calculated and relative density determined from the formula.

Relative density of a liquid

To determine relative density of a liquid using the Archimedes' principle, three measurements are taken:

(i) Weight W_1 of a solid in air

(ii) Weight W_2 of the solid when totally immersed in water

(iii) Weight W_3 of the solid when totally immersed in a liquid whose relative density is to be determined.

$$\text{Relative density of the liquid} = \frac{\text{Mass of liquid}}{\text{Mass of equal volume of water}}$$

$$= \frac{\text{Weight of liquid}}{\text{Weight of equal volume of water}}$$

Since the same solid is used in both the liquid and the water, the volume of water displaced equals volume of liquid displaced. Thus,

Weight of liquid displaced by solid = Upthrust in liquid, and

Weight of water displaced by solid = upthrust in water.

$$\text{Relative density of the liquid} = \frac{\text{Upthrust in the liquid}}{\text{Upthrust in water}}$$

Upthrust in the liquid = $W_1 - W_2$

Upthrust in water = $W_1 - W_2$

Relative density of the liquid = $\frac{W_1 - W_3}{W_1 - W_2}$

Examples

1. A solid mass 0.8 kg suspended by a string is totally immersed in water. If the tension in the string is 4.8 N. calculate

(a) The volume of the solid.

Solution.

$$\begin{aligned}\text{Weight of the solid} &= (0.8 \times 10) \text{ N} \\ &= 8.0 \text{ N}\end{aligned}$$

$$\text{Apparent weight of the solid} = 4.8 \text{ N}$$

$$\begin{aligned}\text{Upthrust} &= (8.0 - 4.8) \text{ N} \\ &= 3.2 \text{ N}\end{aligned}$$

But upthrust = weight of water displaced.

$$\text{Weight of the water displaced} = 3.2 \text{ N}$$

$$\text{Mass of water displaced} = \frac{3.2}{10}$$

$$= 0.32 \text{ kg}$$

$$\text{Volume of water displaced} = \frac{0.32}{1000}$$

$$= 3.2 \times 10^{-4} \text{ m}^3$$

Volume of water displaced = volume of the solid.

$$\text{Volume of the solid} = 3.2 \times 10^{-4} \text{ m}^3$$

- (b) The relative density of the solid.

Solution

$$\text{Relative density of the solid} = \frac{\text{Weight of the solid}}{\text{Upthrust in water}}$$

$$= \frac{8.0}{3.2}$$

$$= 2.5$$

2. In an experiment with a metal cube, the following results were obtained:

Weight of the cube in air = 0.5 N.

Weight of the cube when completely immersed in water = 0.44 N.

Weight of the cube when completely immersed in oil = 0.46 N.

Calculate the relative density of the oil.

Solution

$$\text{Relative density of the oil} = \frac{\text{Upthrust in oil}}{\text{Upthrust in water}}$$

$$= \frac{0.5 - 0.46}{0.5 - 0.44} = \frac{4 \times 10^{-2}}{6 \times 10^{-2}}$$

$$= 0.667$$

Archimedes Principle and Moments.

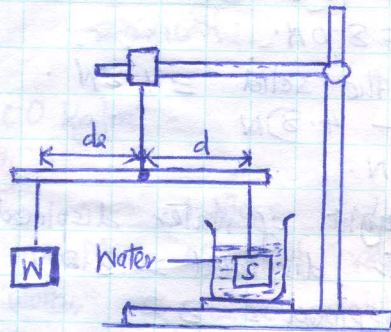
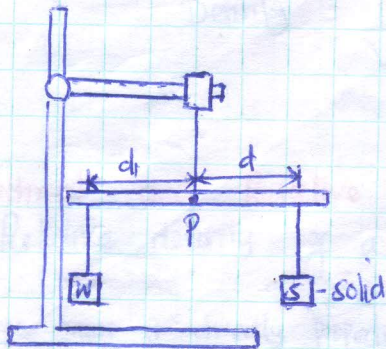
- Relative densities of solids and liquids can also be determined by use of a balanced beam.

Experiment

To determine the relative density of a solid using upthrust and moments.

Apparatus

Metre rule suspended from a clamp, solid S , metal block W , water in a beaker.



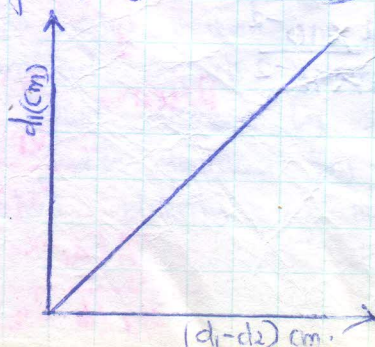
Procedure

- Suspend the metre rule so that it balances at its centre of gravity.
- Suspend the solid S at a distance $d = 30\text{cm}$, then suspend and adjust the position of W such that the rule is balanced.
- Record distance d_1 of W from point of suspension.
- While maintaining the distance d , immerse solid S completely in water, then adjust the position of W to balance the metre rule again.
- Record the new distance d_2 for the metal block W in the table.
- Remove the beaker of water so that the solid S hangs freely in air.
- Increase the distance d to a new value, say 35cm and repeat the procedure above.
- Repeat for four other values of d and complete the table.
- Plot a graph of d_1 against $(d_1 - d_2)$ and determine its slope.

Distance of S in air $d(\text{cm})$	Distance of S in water, $d_1(\text{cm})$	Distance of W , $d_2(\text{cm})$	$d_1 - d_2(\text{cm})$
30			
35			
40			
45			
50			

Observation

- Each time the solid S is immersed in water, the distance d_1 must be adjusted to a small distance d_2 for the metre rule to balance again.
- The graph of d_1 against $(d_1 - d_2)$ is a straight line through the origin.



Calculation

From the principle of moments

$$Wd_1 = W_A d, \text{ where } W_A \text{ is the weight of solid S in air} \dots (1)$$

$$Wd_2 = W_W d, \text{ where } W_W \text{ is the weight of the solid in water} \dots (2)$$

From (1) and (2)

$$W_A = \frac{Wd_1}{d} \text{ and;}$$

$$W_W = \frac{Wd_2}{d}$$

Now, relative density of solid =

$$\frac{W_A}{W_A - W_W}$$

$$= \frac{Wd_1}{d}$$

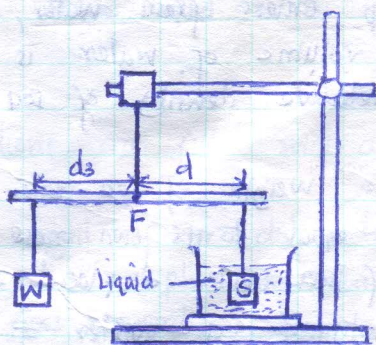
$$\frac{Wd_1 - Wd_2}{d}$$

$$= \frac{d_1}{d_1 - d_2}$$

Thus, the slope of the graph of d_1 versus $(d_1 - d_2)$ represents the relative density of the solid.

Liquids

- To determine the relative density of a liquid, a similar approach to that used in the determination of relative density of a solid is followed.
- In addition to measurements made in the set up, the distance d_3 , at which the lever balances when the solid is immersed in the liquid is noted.



$$Wd_3 = W_L d, \text{ where } W_L \text{ is the apparent weight of the solid in the liquid}$$

$$W_L = \frac{Wd_3}{d} \dots (3)$$

$$\text{Upthrust in water} = W_A - W_W$$

$$= \frac{Wd_1}{d} - \frac{Wd_2}{d}$$

$$= \frac{W}{d} (d_1 - d_2)$$

$$\text{Upthrust in liquid} = W_A - W_L$$

$$= \frac{Wd_1}{d} - \frac{Wd_3}{d}$$

$$= \frac{W}{d} (d_1 - d_3)$$

$$\text{Relative density of liquid} = \frac{\text{upthrust in liquid}}{\text{upthrust in water}}$$

$$= \frac{\frac{W}{d} (d_1 - d_3)}{\frac{W}{d} (d_1 - d_2)} = \frac{d_1 - d_3}{d_1 - d_2}$$

Examples

1. A glass tube of uniform diameter 2.4 cm is weighted to float vertically in a liquid. The length of tube immersed in the liquid is 14 cm. If the density of the liquid is 1.2 g/cm^3 , find the mass of the tube and its contents.

Solution

$$\begin{aligned}\text{Area of cross section of the tube} &= \pi r^2 \\ &= 3.142 \times (1.2)^2 \\ &= 4.52 \text{ cm}^2\end{aligned}$$

$$\frac{32}{7} \times 1.2^2 \times 4.526$$

$$\begin{aligned}\text{Volume of liquid displaced by the tube} &= 4.52 \times 14 \\ &= 63.3 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Mass of liquid displaced by the tube} &= 63.3 \times 1.2 \\ &= 75.96 \text{ g}\end{aligned}$$

$$\begin{aligned}\therefore \text{Mass of tube and contents} &= 0.076 \text{ kg} \\ &= 76.032 \text{ g} \\ &= 0.076036 \text{ kg}\end{aligned}$$

2. A ship of mass 1200 t floats in sea-water. What volume of sea water does it displace? If the ship enters fresh water, what mass of cargo must be unloaded so that the same volume of water is displaced as before? (Density of fresh water = 1000 kg/m^3 , relative density of sea-water = 1.03, $1 \text{ t} = 1000 \text{ kg}$)

Solution

The ship displaces a weight of sea-water equal to its own weight and therefore a mass of sea-water equal to its own mass

$$\text{Mass of sea-water displaced} = 1200 \text{ t}$$

$$\text{Density of sea-water} = 1000 \times 1.03 \text{ kg/m}^3$$

$$\begin{aligned}\text{Volume displaced} &= \frac{\text{mass}}{\text{density}} = \frac{1200 \times 1000}{1000 \times 1.03} \text{ m}^3 \\ &= 1165 \text{ m}^3 \text{ of sea-water.}\end{aligned}$$

The same volume of fresh water has a mass of $1165 \times 1000 = 1165 \text{ t}$.

$$\text{Therefore mass of cargo to be unloaded} = 1200 - 1165 = 35 \text{ t.}$$

3. What volume of brass of density 8.5 g/cm^3 must be attached to a piece of wood of mass 100 g and density 0.2 g/cm^3 so that the two together will just submerge beneath water?

Solution

The brass and wood together will just submerge when the average density of the whole is equal to that of water (1 g/cm^3)

Let the volume of the brass = V

Mass of brass = $8.5V$

Volume of wood = $\frac{100}{0.2} = 500 \text{ cm}^3$

Mass of wood = $100g$

For an average density of 1 g/cm^3 the total mass must be numerically equal to the total volume.

Hence $100 + 8.5V = 500 + V$

$$7.5V = 400$$

$$V = \frac{400}{7.5} = 53.3 \text{ cm}^3 \text{ of brass}$$

4. An ordinary hydrometer of mass $28g$ floats with 3 cm of its stem out of water. The area of cross-section of the stem is 0.75 cm^2 . Find the total volume of the hydrometer and the length of stem above the surface when it floats in a liquid of relative density 1.4 .

Solution

By the law of floatation, the hydrometer displaces its own weight, and hence also its own mass of any liquid.

$\therefore$ Mass of water displaced = $28g$.

Volume of water displaced = 28 cm^3 .

Assuming top of stem to be flat and not rounded.

Volume of stem above water = $0.75 \times 3 = 2.25 \text{ cm}^3$.

Total volume of hydrometer = $28 + 2.25 = 30.25 \text{ cm}^3$

Volume of liquid displaced = $\frac{\text{mass}}{\text{density}} = \frac{28}{1.4} = 20 \text{ cm}^3$.

$\therefore$ Volume of stem above the liquid = $30.25 - 20 = 10.25 \text{ cm}^3$.

hence length of stem above liquid = $\frac{10.25}{0.75} = 13.7 \text{ cm}$.

Applications.

Balloons

Balloons used for meteorological investigations are filled with a gas of low density such as hydrogen or helium.

Due to the low density of the gas, the weight of air displaced by the balloons is greater than weight of the gas in the balloon plus the balloon fabric.

When released the balloon therefore rises upwards. It gains altitude at some height where the density of air is less than that at the ground, the upthrust on the balloon is equal to its weight.

- Resultant force on the balloon is zero and the balloon stops rising, but may drift sideways in the direction of wind.

Ships

Since steel is denser than water, solid steel will sink in water. However a ship which is made of steel is hollow and the volume of water displaced is large. The upthrust exerted by the water on the ship is thus large.

- Like other floating bodies, the ship sinks to some level so that the weight of water displaced (upthrust) is equal to its weight.

When the ship is loaded it sinks more into the water.

Let the volume of the brass = V

Mass of brass = $8.5V$

Volume of wood = $\frac{100}{0.2} = 500 \text{ cm}^3$

Mass of wood = $100g$

For an average density of 1 g/cm^3 the total mass must be numerically equal to the total volume.

Hence $100 + 8.5V = 500 + V$

$$7.5V = 400$$

$$V = \frac{400}{7.5} = 53.3 \text{ cm}^3 \text{ of brass}$$

4. An ordinary hydrometer of mass $23g$ floats with $3cm$ of its stem out of water. The area of cross-section of the stem is 0.75 cm^2 . Find the total volume of the hydrometer and the length of stem above the surface when it floats in a liquid of relative density 1.4 .

Solution

By the law of floatation, the hydrometer displaces its own weight, and hence also its own mass of any liquid.

$\therefore$ Mass of water displaced = $23g$

Volume of water displaced = 23 cm^3

Assuming top of stem to be flat and not rounded.

Volume of stem above water = $0.75 \times 3 = 2.25 \text{ cm}^3$

Total volume of hydrometer = $23 + 2.25 = 25.25 \text{ cm}^3$

Volume of liquid displaced = $\frac{\text{mass}}{\text{density}} = \frac{23}{1.4} = 16.43 \text{ cm}^3$

$\therefore$ Volume of stem above the liquid = $25.25 - 16.43 = 8.82 \text{ cm}^3$

hence length of stem above liquid = $\frac{8.82}{0.75} = 11.76 \text{ cm}$

Applications

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Balloons used for meteorological investigations are filled with a gas of low density such as hydrogen or helium.

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- Like other floating bodies, the ship sinks to some level so that the weight of water displaced (upthrust) is equal to its weight.

When the ship is loaded, it sinks more into the water.

Submarines

- A submarine is designed so that it can float or sink below the surface of water. It is fitted with large flotation tanks which can be filled with water or air, hence varying its weight. If the submarine is required to sink, the tanks are filled with water. The weight of the submarine thus becomes greater than the upthrust due to the water on it.
- When the submarine is required to float, compressed air is forced into the flotation tanks driving the water out. Its weight becomes less than upthrust on it, making it rise to the surface.

Applications of Archimedes' principle and Relative Density

The hydrometer

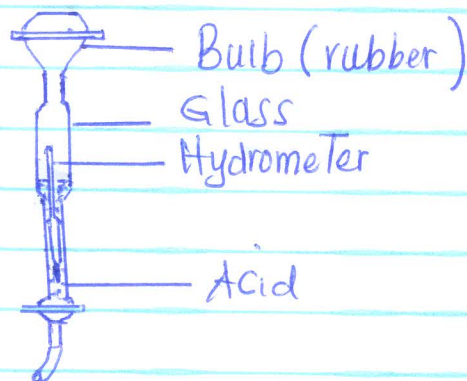
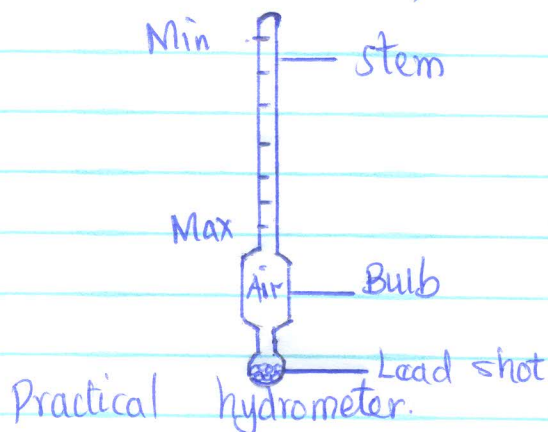
- A hydrometer is an instrument used to directly determine the densities or relative densities of liquids.
- A hydrometer uses the law of flotation in its operation.
- The main features of a hydrometer are:

(i) Wide bulb containing air

- The bulb is made wide so that it can displace a large volume of liquid that provides a sufficient upthrust to keep the hydrometer floating.
- The volume of the bulb determines the density range to be measured by the hydrometer.
- Lead shot is waxed or glued to the bottom of the bulb to make the hydrometer float upright.

(ii) A narrow graduated hollow stem

The narrower the stem, the more sensitive it is.

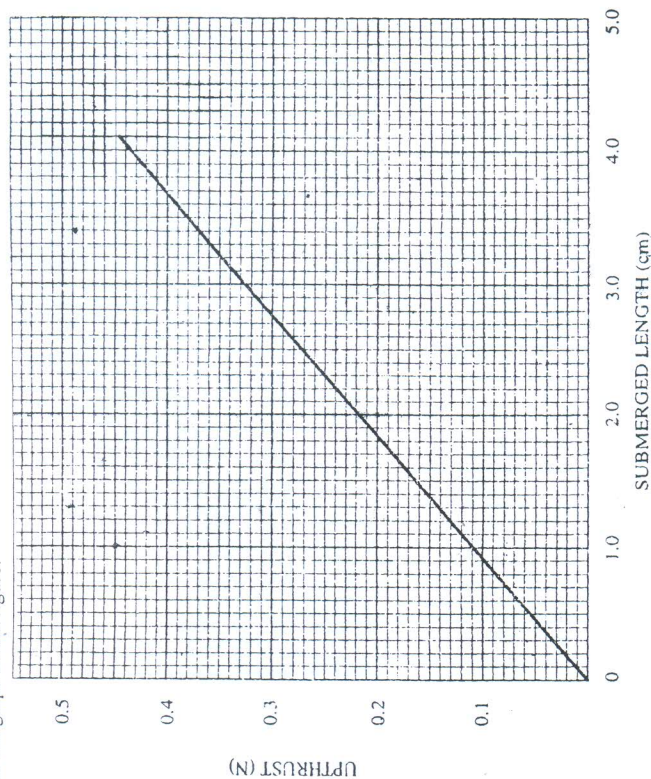


CHAPTER 3: FLOATING AND SINKING

1. State the Archimedes principle. (1mark)
2. State two conditions for a body to float on water. (1mark)

3. (a) A test tube of uniform cross section is loaded so that it can float upright in water. With the aid of a labeled diagram describe how the test tube may be calibrated to measure density (3marks)

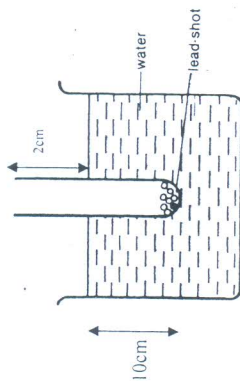
(b) In an experiment to determine the density of a liquid, a uniform metal cylinder of cross sectional area 6.2 cm^2 and length 4.5 cm was hung from a spring balance and lowered gradually into the liquid. The upthrust was determined for various submerged lengths. The results obtained are shown on the graph in the figure.



Use the graph to determine

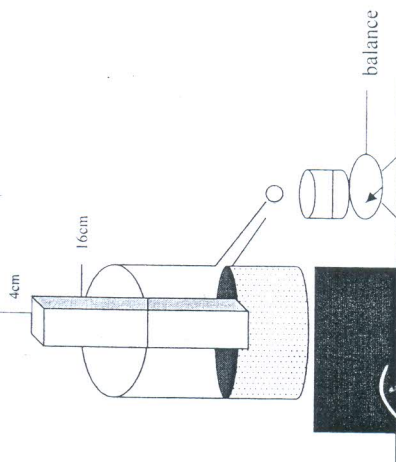
- (i) the value of the up thrust when the cylinder is fully submerged (2mark)
- (ii) the density of the liquid (3marks)

4(a) A simple hydrometer is set up with a test tube of mass 10 g and length 12 cm with a flat base and partly filled with lead shots. The test tube has uniform cross-sectional area of 2.0 cm^2 and 10 cm of its length under water as shown in the diagram



- (i) taking the density of water as 1000 kg/m^3 , Calculate the mass of the lead shots in the tube (3marks)
- (ii) The mass of the lead shots to be added if it has to displace an equal volume of a liquid of density 1.25 g/cm^3 . (3marks)

5. The figure shows a block with a graduated side and dimensions $4 \text{ cm} \times 4 \text{ cm} \times 16 \text{ cm}$, just about to be lowered into a liquid contained in an overflow can



During an experiment with this set up the following information was recorded.
 -the block floated with three quarters of it submerged
 -initial reading of balance = 0 g
 -final reading of balance = 154 g

Use the information to determine the density of the

(3marks)

(i) block

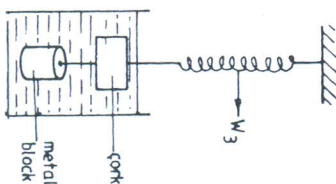
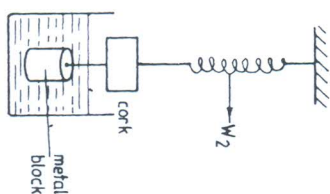
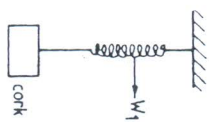
(3marks)

(ii) liquid.

(3marks)

6. A block of metal is suspended from the lower end of a spring balance and is then gradually lowered into water until its upper end is some distance below the surface. Describe the changes observed in the readings of the spring balance during the process.

(3marks)



Weight of cork in air = W_1

Weight of cork in air and metal in water = W_2

Weight of both cork and metal in water = W_3

(i) write an expression for the up thrust on the cork in water

(2marks)

(ii) derive an expression for the R.D of the cork

(2marks)

(iii) Given that $W_1 = 0.08\text{N}$, $W_2 = 0.60\text{N}$ and $W_3 = 0.28\text{N}$ calculate the relative density of the cork (3mk)

(iv) A piece of wax of mass 380g and volume 400cm^3 is kept under water by tying it with a thin thread to a piece of metal. Determine the tension in the thread. (3marks)

7. A block of wood measuring 2m by 0.5m by 0.8m floats in water of density 1000kg/m^3 with the longest side vertical. If the length of the block in water is 1.2m calculate

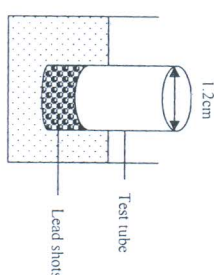
(i) the up thrust on the block

(3marks)

(ii) the force needed to just make the block fully submerged

(1mark)

8. A student used a test tube of diameter 1.2cm and mass 5g to make a hydrometer. To make it float upright he added some lead shots of mass m g as shown in the diagram. He then placed the hydrometer in a liquid of density 0.8g/cm^3 . It sunk to a depth of 6cm. Calculate the mass of the shots added. (3marks)



9. A hot air balloon is tethered to the ground on a windless day. The envelope of the balloon contains 140m^3 of hot air of density 0.7kg/m^3 . The mass of the balloon (not including the hot air) is 380kg. The density of the surrounding air is 1.3kg/m^3

(i) Calculate the tension in the rope holding the balloon to the ground. (4marks)

(ii) Calculate the acceleration with which the balloon begins to rise when the rope is cut and is released. (3marks)

10. A 200kg wooden block floating with 0.2 of its volume above the water surface. What is the minimum volume of an object of density 0.79g/cm^3 which must be attached to the underside of the block to completely submerge? (3marks)

11. A balloon weighs 10N and it has a gas capacity of 2m^3 . The gas in the balloon has a density of 0.1kg/m^3 . If the density of the air is 1.3kg/m^3 calculate the resultant force on the balloon when it is floating in air. (3marks)

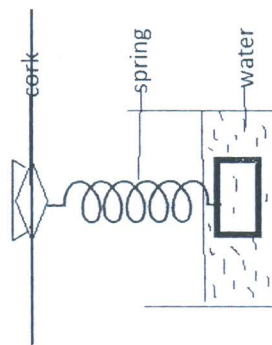
12. A bar of mass 180g and density 8.5g/cm^3 is attached to a piece of wood of mass 70g and both are totally immersed in water. The apparent mass is 120g, calculate the density of the wood. (3mark)

13. Steel has a density of 8g/cm^3 and is attached to a piece of cork of mass 50g and density 0.8g/cm^3 such that the two will together just sink below water. Calculate the volume of the steel which must be attached. (3marks)

14. (i). A hydrometer of mass 10g is placed in water of density 1g/cm^3 . Calculate how much length will sink in the water if the cross-section area of the uniform tube is 1cm^2 . (3marks)

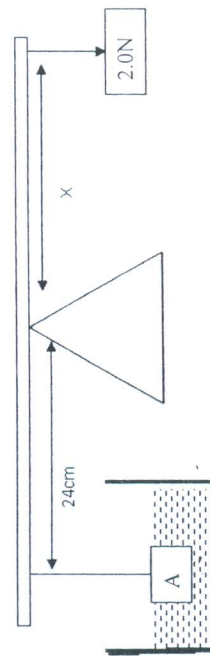
(ii). If the same hydrometer is laced in a liquid of density 3g/cm^3 how much of it will sink (2marks)

15. The diagram shows a steel cube of side 5cm hanging from a cork. The spring shown is of constant 20N/m . Calculate the extension of the spring. (density of steel $= 800\text{kg/m}^3$) (3marks)

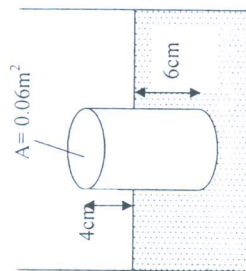


16. A block of wood of mass 100kg and density 200kg/m^3 floats on water of density 1000kg/m^3 . Calculate the maximum weight that can be placed on the block before it is fully submerged. (3marks)

17. The figure shows a block A that weighs 2N in air and density of 2.0g/cm^3 . If the bar is pivoted at its center and is in equilibrium find the distance X. Density of water is 1g/cm^3 . (3marks)



18. The diagram below shows a wooden log 10m long, density 800kg/m^3 and cross-sectional area 0.06m^2 floating in water of density 1g/cm^3 while partially submerged.



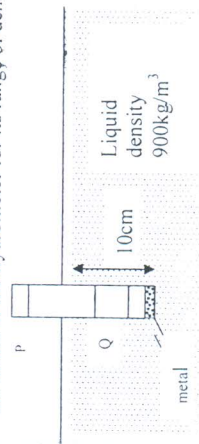
(i) Determine the weight of the block (3marks)

(ii) The up thrust on the block (3marks)

(iii) The minimum weight that can be placed on the block to just make it fully submerged. (3mk)

19. Describe an experiment to demonstrate the law of floatation (5marks)

(b) The figure shows a hydrometer used to measure the density of liquids over a range of 800kg/m^3 to 1200kg/m^3 . The hydrometer has a cross-sectional area of 2.0cm^2 and height 15cm . Marks P and Q are the limits of the hydrometer for its range of densities.



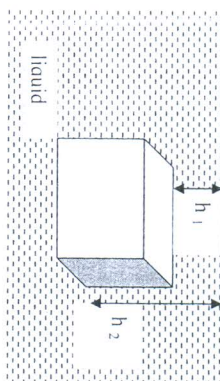
(i) Explain why the piece of metal is inserted into the base of the wood (1mark)

(ii) Which of the two densities 800kg/m^3 and 1200kg/m^3 does mark Q represent? Give a reason (2marks)

- (iii) When immersed in a liquid of density 900kg/m^3 the hydrometer sinks to a depth of 10cm as shown in the fig. Calculate the vol of the liquid displaced by the hydrometer. (3marks)

- (iv) Calculate the mass of the hydrometer (2marks)

20. A block of rectangular cross section area $A\text{m}^2$ is immersed in a liquid of density $\rho\text{kg/m}^3$. The top and bottom surface of the block are respectively 1m and 2m below the surface as shown below



- (i) What is the downward force due to the liquid on the top surface of the block (3marks)
- (ii) What is the upward force due to the liquid on the lower surface of the block (3marks)
- (iii) Use the results in i and ii to illustrate Archimedes principle (2marks)
- (iv) Under what condition will the net force be equal to the weight of the immersed block (1mark)

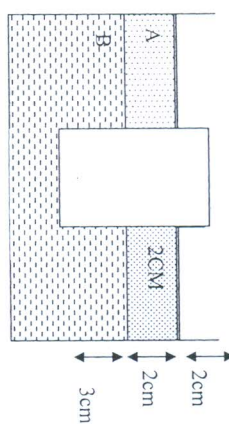
21. When it was first proposed to build ships made of iron many people laughed at the suggestion. Owing to the fact that a piece of iron sinks when placed in water they held the view that iron ships would be a failure. Explain why it is possible for ships to float on water. (2marks)

(b) It is reported that on one occasion King Hiero of Syracuse asked Archimedes to ascertain if a gold crown made by the court goldsmith had been fraudulently alloyed with silver. The problem remained unsolved until Archimedes could find a way of measuring the volume of the crown. Tradition has it that the solution occurred to him one day at the baths. On immersing himself in a bath full of the brim he suddenly realized that the volume of water which spilled over was equal to his own volume. The eureka can is an illusion to the legend that in his excitement at the discovery, Archimedes ran naked through the streets shouting "Eureka" (=I have found it).

The following example illustrates the kind of calculation which Archimedes may have made to incriminate the goldsmith

A lump of gold-silver alloy has a mass of 350.8g and a volume of 20cm^3 . Assuming that no change in volume occurs when gold is alloyed with silver, find the mass of silver in the sample. (density of gold = 19.3g/cm^3 of silver = 10.5g/cm^3). (3marks)

22. The figure represents a block of uniform cross-sectional area 6.0cm^2 floating on two liquids A and B. The length of the block in each liquid is shown.

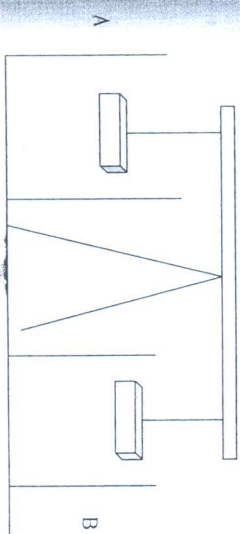


- Give that the density of liquid A is 800kg/m^3 and that of liquid B is 1000kg/m^3 determine the
- (i) weight of liquid A displaced (3marks)

- (ii) weight of liquid B displaced (3marks)

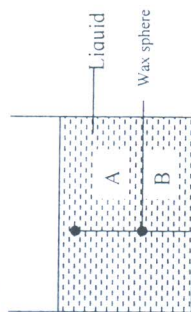
- (iii) Density of the block. (4marks)

23. The figure shows a uniform bar in equilibrium



When water is added into beakers A and B until the weights are submerged it is observed that the bar tips towards B. Explain this observation. (2marks)

24.(a) Fig shows two spheres made of wax each of mass 0.1kg held in a liquid by strings A and B



If the up thrust on each sphere is 1.05N determine the tension in each string. (3marks)

(b) A block of glass of mass 250g floats in mercury. What volume of the glass lies under the surface of the mercury? Density of mercury is 13600kg/m^3 . (3marks)

25.(a). Explain the following observation. ice cubes float on water and solid benzene sinks in liquid benzene. (2marks)

(b) You are provided with the following
An overflow can, a beaker, a spring balance, a metal block, water and string
Describe an experiment to verify Archimedes principle (4marks)

(c) A block of wood weighing 2.0N is held under water by a string attached to the bottom of a container. The tension in the string is 0.5N. Determine the density of the wood. (3marks)

26.(a) An object of mass m has a weight w_1 in air and w_2 in water. Suggest a reason why w_1 is greater than w_2 . (1 mark)

(b) The diagram in the figure shows a block of wood floating to a depth of 0.18m in water.



Determine

- The mass of the wood (3marks)
 - The density of the wood (3marks)
- (b) Determine the number of frogs each of mass 500g which will make the block of wood be just submerged if they sit on the block. (3marks)

27. An object of mass 120g half immersed in water displaced a volume of 20cm^3 . Calculate the density of the object. (2marks)

28. A right angled solid of dimensions 0.02m by 0.02m by 0.2m and density 2700kg/m^3 is supported inside kerosene of density 800kg/m^3 by a thread which is attached to the spring balance. The long side is vertical and the upper side is 0.1m below the surface of kerosene. Calculate the force due to the liquid on

- The lower surface of the solid (4marks)
- The upper surface of the solid (2marks)
- Calculate the up thrust on the solid (4marks)
- Hence or otherwise determine the reading on the spring balance. (3marks)